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LETTER TO THE EDITOR

Exact solution of the $\mathbb{Z}_n \times \mathbb{Z}_n$ symmetric generalisation of Baxter's eight-vertex model

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Abstract. The generalisation of the quantum inverse scattering method is used to diagonalise the transfer matrix of Belavin's $\mathbb{Z}_n \times \mathbb{Z}_n$ symmetric model. The eigenvalues and Bethe ansatz equations are obtained.

Consider a two-dimensional square lattice with the vertices of Boltzmann weight $S(z)_{i,j}^{k,l}$, which satisfy the Yang-Baxter relation (Yang 1967, Karowski *et al* 1977, Zamolodchikov 1977, Zamolodchikov and Zamolodchikov 1978)

$$S(z_1-z_2)_{i_1,i_2}^{k_1,k_2} S(z_1-z_3)_{k_1,i_3}^{j_1,k_3} S(z_2-z_3)_{k_2,k_3}^{j_2,j_3}$$
$$= S(z_2-z_3)_{i_2,i_3}^{k_2,k_3} S(z_1-z_3)_{i_1,k_3}^{k_1,j_3} S(z_1-z_2)_{k_1,k_2}^{j_1,j_2} \tag{1}$$

where the double k indices mean summations over $0, 1, \ldots, n-1$. One possible n-state generalisation of Baxter's eight-vertex model (Baxter 1972) is the $\mathbb{Z}_n \times \mathbb{Z}_n$ symmetric model of Belavin (1981) which has the Boltzmann weight $S(z)$ satisfying the Yang-Baxter relation (1) (Bovier 1983, Cherednik 1982, Tracy 1985). Richey and Tracy (1986) have shown that the Belavin parametrisation of Boltzmann weight $S(z)$ can be written in product form

$$'S(z)_{i,j}^{k,l} = \begin{cases} 0 & \text{if } i+j \neq k+l \\ \dfrac{h(z)\theta^{(i-j)}(z+w)}{\theta^{(i-k)}(w)\theta^{(k-j)}(z)} & \text{if } i+j=k+l;\ \text{mod } n \end{cases}$$

with

$$h(z) = \prod_{i=0}^{n-1} \theta^{(i)}(z) \left(\prod_{i=1}^{n-1} \theta^{(i)}(0) \right)^{-1}$$

where $\theta^{(i)}(z)$ represents the theta functions of rational characteristics $1/2 - i/n$, $1/2$ (Richey and Tracy 1986).

Recently Jimbo *et al* (1987a, b) derived the vertex-IRF (interaction round a face) correspondence transforming Belavin's $\mathbb{Z}_n \times \mathbb{Z}_n$ symmetric model into an IRF model. This IRF model can, however, be thought of as a multicomponent generalisation, using elliptic functions, of the six-vertex model. We expect the model is exactly solvable in the sense that the eigenstates of the transfer matrix can be exactly constructed.

An algebraisation of the Bethe ansatz method (Baxter 1973) constructing exactly the eigenstates of the eight-vertex model was found by Takhtadzhan and Faddeev (1979). The multicomponent generalisation of the six-vertex model was diagonalised by Babelon *et al* (1982) and Schultz (1983) using the quantum inverse scattering method (QISM) (Sklyanin and Faddeev 1978, Faddeev 1981, Thacker 1981). In this letter we treat the $\mathbb{Z}_n \times \mathbb{Z}_n$ symmetric model of Belavin by using QISM and diagonalise the transfer matrix of the model. In our method the vertex-IRF correspondence given by Jimbo *et al* (1987a, b) is available to construct the eigenstates of the transfer matrix of the model. Here we rewrite the vertex-IRF correspondence in two forms

$$'S(z_1-z_2)M_a^{\nu}(z_1)\otimes M_{a+\hat{\mu}}^{\mu}(z_2)=M_{a+\hat{\mu}}^{\mu'}(z_1)\otimes M_{a+\hat{\mu}-\hat{\mu}'}^{\nu'}(z_2)r_1(a|z_1-z_2)_{\mu',\nu'}^{\mu,\nu} \tag{2a}$$

and (Jimbo *et al* 1987a, b)

$$'S(z_1-z_2)M_a^{\mu}(z_1)\otimes M_{a+\hat{\mu}}^{\nu}(z_2)=M_{a+\hat{\nu}'}^{\mu'}(z_1)\otimes M_a^{\nu'}(z_2)r_2(a|z_1-z_2)_{\mu',\nu'}^{\mu,\nu} \tag{2b}$$

where the double indices μ' and ν' mean the summations over $0, 1, \ldots, n-1$ and

$$M_a^{\mu}(z)=\phi_a^{\mu}(z)\prod_{i=0}^{n-1} g_{i\mu}^{-1}(a) \qquad \mu=0,1,\ldots,n-1$$

$$g_{i\mu}(a)=\begin{cases}1 & \text{as } i\geqslant\mu\\ h(wa^{0,\mu}+s^0-s^{\mu}) & \text{as } i=0\\ h(wa^{i,\mu}-w+s^i-s^{\mu}) & \text{as } 0<i<\mu.\end{cases} \tag{3}$$

The r_1 and r_2 have the form

$$r_1(z)_{\mu',\nu'}^{\mu,\nu}=\alpha_1^{\mu\nu}(a|z)\delta_{\mu'\nu}\delta_{\nu'\mu}+\beta_1^{\mu\nu}(a|z)\delta_{\mu\mu'}\delta_{\nu\nu'} \tag{4a}$$

$$r_2(z)_{\mu',\nu'}^{\mu,\nu}=\alpha_2^{\mu\nu}(a|z)\delta_{\mu'\mu}\delta_{\nu'\nu}+\beta_2^{\mu\nu}(a|z)\delta_{\mu\nu'}\delta_{\mu'\nu} \tag{4b}$$

$$\alpha_1^{\mu\nu}(a|z)=\frac{h(z)}{h(w)}\frac{h(s^{\mu}-s^{\nu}+wa^{\mu\nu})}{h(s^{\mu}-s^{\nu}+wa^{\mu\nu}+w)}\prod_{i=0}^{n-1}\frac{g_{i\mu}(a+\hat{\mu}-\hat{\nu})}{g_{i\mu}(a+\hat{\mu})}\prod_{i=0}^{n-1}\frac{g_{i\nu}(a+\hat{\mu})}{g_{i\nu}(a)} \tag{5a}$$

$$\alpha_2^{\mu\nu}(a|z)=\frac{h(z)}{h(w)}\frac{h(s^{\nu}-s^{\mu}+w(a-\hat{\nu})^{\nu,\mu})}{h(s^{\nu}-s^{\mu}+w(a-\hat{\nu})^{\nu,\mu}+w)}\prod_{i=0}^{n-1}\frac{g_{i\nu}(a)}{g_{i\nu}(a+\hat{\mu})}\prod_{i=0}^{n-1}\frac{g_{i\mu}(a+\hat{\nu})}{g_{i\mu}(a)} \tag{5b}$$

$$\beta_1^{\mu\nu}(a|z)=\frac{h(s^{\mu}-s^{\nu}+wa^{\mu,\nu}+w+z)}{h(s^{\mu}-s^{\nu}+wa^{\mu,\nu}+w)} \tag{5c}$$

$$\beta_2^{\mu\nu}(a|z)=\beta_1^{\nu\mu}(a-\hat{\nu}|z). \tag{5d}$$

Equations (2*a*) and (2*b*) can be held for $\mu=\nu$ if we impose $(a-\hat{\nu})^{\mu\mu}=0$ and $a^{\mu\mu}=0$. The column vector $\phi_a^{\mu}(z)$ and some notations $\hat{\mu}$ and $a^{\mu\nu}$ are given by Jimbo *et al* (1987a, b). The s^{μ} are arbitrary complex parameters. Throughout this letter $a, b\in\Sigma_{\mu=0}^{n-1}\mathbb{C}\Lambda_{\mu}$ where the Λ_{μ} are the fundamental weights of the affine Lie algebra $A_{n-1}^{(1)}$, and $\hat{\mu}=\Lambda_{\mu+1}-\Lambda_{\mu}$, $\mu=0,1,\ldots,n-1$ and $\Lambda_0=\Lambda_n$. The $\phi_a^{\mu}(z)$ are linearly independent and we can define the row vectors $M_a^{-\mu}(z)$ by

$$\begin{aligned}&M_a^{-\mu}(z)M_a^{\nu}(z)=\delta_{\mu,\nu}\\ &\sum_{\mu=0}^{n-1}M_a^{\mu}(z)_i M_a^{-\mu}(z)_j=\delta_{i,j}.\end{aligned} \tag{6}$$

From (2*a*) we have the vertex-IRF correspondence for the $M_a^{-\mu}(z)$

$$M_a^{-\mu}(z_1)\otimes M_{a-\hat{\mu}}^{-\nu}(z_2)'S(z_1-z_2)=r_1(a-\hat{\mu}'|z_1-z_2)_{\mu,\nu}^{\mu',\nu'}M_{a-\hat{\mu}'}^{-\nu'}(z_1)\otimes M_a^{-\mu'}(z_2) \tag{7}$$

where the double indices μ', ν' mean the summations over $0, 1, \ldots, n-1$. Equation (2b) was given by Jimbo *et al* (1987a, b) and the same method can be used to give equation (2a).

The elements of the r_1 and r_2 matrices of (4a) and (4b) are the Boltzmann weights in the IRF model. With the help of equations (2a) and (2b) and from (1) we can find the Yang–Baxter relations for r_1 and r_2. Here only the Yang–Baxter relation for the r_2 is used; it is

$$r_2(a+\hat{\mu}_3|z_1-z_2)^{\alpha_2,\alpha_1}_{\mu_2,\mu_1} r_2(a|z_1-z_3)^{\beta_2,\alpha_3}_{\alpha_2,\mu_3} r_2(a+\hat{\beta}_2|z_2-z_3)^{\beta_1,\beta_3}_{\alpha_1,\alpha_3}$$
$$= r_2(a|z_2-z_3)^{\alpha_1,\alpha_3}_{\mu_1,\mu_3} r_2(a+\hat{\alpha}_1|z_1-z_3)^{\alpha_2,\beta_3}_{\mu_2,\alpha_3} r_2(a|z_1-z_2)^{\beta_2,\beta_1}_{\alpha_2,\alpha_1} \tag{8}$$

where the double indices α mean the summations over $0, 1, \ldots, n-1$.

From $'S$ we construct the monodromy matrix for a line of N sites

$$T_N(z) = {}'S(z-z_N^0)\cdots {}'S(z-z_2^0)\,{}'S(z-z_1^0) \tag{9}$$

where

$$'S(z-z_s^0)_{i,k} = \sum_{i,j=0}^{n-1} {}'S(z-z_s^0)^{k,l}_{i,j} E_{j,l}.$$

$E_{j,l}$ is an $n\times n$ matrix with $(E_{j,l})_{i,k} = \delta_{i,j}\delta_{k,l}$. The z_s^0 are a set of constants attached to the sites on the line.

This matrix satisfies the remarkable permutation relation obtained by using (1)

$$'S(z_1-z_2)^{k_1,k_2}_{i_1,i_2} T_N(z_1)_{k_1,j_1} T_N(z_2)_{k_2,j_2} = T_N(z_2)_{i_2,k_2} T_N(z_1)_{i_1,k_1}\, {}'S(z_1-z_2)^{j_1,j_2}_{k_1,k_2} \tag{10}$$

and consequently the transfer matrix

$$t(z) = \sum_{i=0}^{n-1} T_N(z)_{i,i}$$

satisfies

$$[t(z), t(z')] = 0. \tag{11}$$

In order to construct the eigenstates of the transfer matrix $t(z)$ we should replace $T_N(z)$ by $T_{b,a}(z)$ gauge equivalent to $T_N(z)$ as the method of Takhtadzhan and Faddeev (1979) for studying Baxter's eight-vertex model

$$T^{\mu,\nu}_{b,a}(z) = M_b^{-\mu}(z) T_N(z) M_a^{\nu}(z)$$
$$= \begin{cases} A_{b,a}(z) & \text{as } \mu=\nu=0 \\ B^{\nu}_{b,a}(z) & \text{as } \mu=0,\ \nu>0 \\ C^{\mu}_{b,a}(z) & \text{as } \mu>0,\ \nu=0 \\ D^{\mu,\nu}(z) & \text{as } \mu>0,\ \nu>0 \end{cases} \tag{12a}$$

$$t(z) = A_{a,a}(z) + \sum_{\mu=1}^{n-1} D^{\mu,\mu}_{a,a}(z) \qquad \text{for all } a \in \sum_{\mu=0}^{n-1} \mathbb{C}\Lambda_\mu. \tag{12b}$$

With the help of (2a), (2b) and (7) and using (10) we can obtain all permutation relations among the operators A, B, C and D. We write out some of them, which play a fundamental role for the construction of the eigenstates of the model. They are

$$A_{b,a}(z_1)B^{\beta}_{b+\hat{0},a+\hat{\beta}_0}(z_2)$$

$$=\frac{h(w+z_2-z_1)}{h(z_2-z_1)}B^{\beta}_{b,a+\hat{\beta}_0-\hat{0}}(z_2)A_{b+\hat{0},a+\hat{\beta}}(z_1)$$

$$-\frac{h(w)\beta_1^{\beta,0}(a+\hat{\beta}_0-\hat{\beta}|z_1-z_2)}{h(z_2-z_1)}B^{\beta}_{b,a+\hat{\beta}_0-\hat{0}}(z_1)(A_{b+\hat{0},a+\hat{\beta}}(z_2) \tag{13a}$$

$$D^{\alpha\beta_0}_{b,a}(z_1)B^{\beta}_{b+\hat{0},a+\hat{\beta}_0}(z_2)$$

$$=\frac{h(w)}{h(z_1-z_2)}B^{\beta'}_{b+\hat{0}-\hat{\alpha},a}(z_2)D^{\alpha,\beta_0'}_{b+\hat{0},a+\hat{\beta}'}(z_1)r_2(a|z_1-z_2)^{\beta_0,\beta}_{\beta_0',\beta}$$

$$-\frac{\beta_1^{\alpha,0}(b-\hat{\alpha}+\hat{0}|z_1-z_2)h(w)}{h(z_1-z_2)}B^{\beta_0}_{b+\hat{0}-\hat{\alpha},a}(z_1)D^{\alpha,\beta}_{b+\hat{0},a+\hat{\beta}_0}(z_2) \tag{13b}$$

$$B^{\beta_0}_{b,a}(z_1)B^{\beta}_{b+\hat{0},a+\hat{\beta}_0}(z_2)$$

$$=\frac{h(w)}{h(z_1-z_2+w)}B^{\beta_0'}_{b,a}(z_2)B^{\beta'}_{b+\hat{0},a+\hat{\beta}_0'}(z_1)r_2(a|z_1-z_2)^{\beta_0,\beta}_{\beta',\beta_0'} \tag{13c}$$

where the double indices β_0' and β' mean the summations over $1, 2, \ldots, n-1$. In the following we construct the eigenstates of the transfer matrix $t(z)$. Define the vacuum state

$$|0, a\rangle = M^0_{a+N\hat{0}}(z^0_N)\otimes\ldots\otimes M^0_{a+2\hat{0}}(z^0_2)\otimes M^0_{a+\hat{0}}(z^0_1). \tag{14}$$

Using equations (2a), (9) and (12a) we have

$$A_{a+N\hat{0},a}(z)|0, a\rangle = \prod_{r=1}^{N}\frac{h(w+z-z_r^0)}{h(w)}|0, a-\hat{0}\rangle \tag{15a}$$

$$D^{\mu,\nu}_{a+N\hat{0},a}(z)|0, a\rangle = \delta_{\mu,\nu}\prod_{r=1}^{N}\frac{h(z-z_r^0)}{h(w)}|0, a-\hat{\mu}\rangle \tag{15b}$$

$$B^{\nu}_{a+N\hat{0},a}(z)|0, a\rangle \neq 0. \tag{15c}$$

From (13a)–(15c) it is obvious that we should examine the state

$$|z_1^1, z_2^1, \ldots, z_{p_1}^1, \boldsymbol{\theta}\rangle$$

$$=\sum_a \exp[\mathrm{i}\boldsymbol{\theta}\cdot(a+\hat{\beta}_0)]f_{\beta_{p_1}\ldots\beta_1}B^{\beta_1}_{a+\hat{0},a+\hat{\beta}_0}(z_1^1)B^{\beta_2}_{a+2\hat{0},a+\hat{\beta}_0+\hat{\beta}_1}(z_2^1)\ldots$$

$$\times B^{\beta_{p_1}}_{a+p_1\hat{0},a+\hat{\beta}_0+\ldots+\hat{\beta}_{p_1-1}}(z^1_{p_1})\left|0, a+\sum_{i=0}^{p_1-1}\hat{\beta}_i\right\rangle \tag{16}$$

with the f defined by

$$\sum_{k=1}^{n-1}T^{k,k}_{p_1}(z)f = t_1(z)f \tag{17}$$

where

$$T^{k_0,k_1}_{p_1}(z)^{\beta_{p_1}\ldots\beta_1}_{\beta'_{p_1}\ldots\beta'_1} = \exp(\mathrm{i}\boldsymbol{\theta}\cdot\hat{k}_0)\prod_{s=1}^{p_1}\frac{h(w)}{h(w+z-z_s^1)}r_2\left(a+\sum_{i=1}^{p_1-1}\hat{\beta}'_i|z-z^1_{p_1}\right)^{k_{p_1},\beta_{p_1}}_{k_0,\beta'_{p_1}}\ldots$$

$$\times r_2(a+\hat{\beta}'_1|z-z_2^1)^{k_2,\beta_2}_{k_3,\beta'_2}r_2(a|z-z_1^1)^{k_1,\beta_1}_{k_2,\beta'_1} \tag{18}$$

where all $k>0$ and $\beta'>0$. $\boldsymbol{\theta}$ is an n-dimensional constant vector. The summation with respect to a is over all $a \in \Sigma\, \mathbb{Z}\hat{\mu}$.

In the following we will show that the state (16) is the eigenstate of the transfer matrix $t(z)$ of (12b) only as $p_1 = (1-1/n)N$.

Step 1. Since the individual vertices associated with the Boltzmann weight ${}'S_{i,j}^{k,l}$ and $r_{\mu,\nu}^{\mu',\nu'}$ obey a conservation law, or $i+j=k+l$ and $\mu+\nu=\mu'+\nu'$, we can impose the conserved quantities p_i, $i=1$ to $n-1$, on the state (16) as follows: there are $N_i = p_i - p_{i+1}$ of the β equal to $i(i=1,\ldots,n-2)$ and $N_{n-1}=p_{n-1}$ of the β' equal to $n-1$ in the set $(\beta_1, \beta_2, \ldots, \beta_{p_1})$. A similar conserved condition is discussed by Schultz (1983) for a multicomponent generalisation of the six-vertex model and by Zhou (1988) for the quantum non-linear Schrödinger model.

Step 2. From (8) and (13c) we can verify that the state is a symmetric function of $z_1^1, z_2^1, \ldots, z_{p_1}^1$.

Step 3. We apply to the state (16) the $A_{a,a}(z)$ and $D_{a,a}^{\beta_0,\beta_0}(z)$. Using the permutation relations (13a) and (13b) we commute the A and D with the B to $|0, a+\Sigma\, \hat{\beta}_i\rangle$. With the help of (15$a$) and (15$b$) and steps 1 and 2 we can find that only as $p_i = (1-i/n)N$, $i=1,\ldots,n-1$, is the state the eigenstate of the transfer matrix $t(z)$ of (12b). The eigenvalue is

$$t_0(z) = \prod_{s=1}^{p_1} \frac{h(w+z_s^1-z)}{h(z_s^1-z)} \prod_{r=1}^{N} \frac{h(w+z-z_r^0)}{h(w)} \exp(\mathrm{i}\boldsymbol{\theta}\cdot\hat{0})$$
$$+ \prod_{s=1}^{p_1} \frac{h(w+z-z_s^1)}{h(z-z_s^1)} \prod_{r=1}^{N} \frac{h(z-z_r^0)}{h(w)} t_1(z) \tag{19}$$

and the z_s^1 satisfy

$$t_0(z_u^1) = 0 \qquad u=1,\ldots,p_1. \tag{20}$$

Step 4. The $t_1(z)$ can be obtained by solving equation (17). With the help of (8) and again using QISM we can find $t_1(z)$ and f of equation (17). The same method can be found in Schultz (1983), Pu and Zhou (1987) and Fan *et al* (1988). Here we only write the final results in the following:

$$t_\mu(z) = \prod_{s=1}^{p_{\mu+1}} \frac{h(w+z_s^{\mu+1}-z)}{h(z_s^{\mu+1}-z)} \exp(\mathrm{i}\boldsymbol{\theta}\cdot\hat{\mu})$$
$$+ \prod_{s=1}^{p_{\mu+1}} \frac{h(w+z-z_s^{\mu+1})}{h(z-z_s^{\mu+1})} \prod_{r=1}^{p_\mu} \frac{h(z-z_r^\mu)}{h(w+z-z_r^\mu)} t_{\mu+1}(z) \qquad \mu=1,\ldots,n-2 \tag{21a}$$

$$t_{n-1}(z) = \exp[\mathrm{i}\boldsymbol{\theta}\cdot(\widehat{n-1})] \tag{21b}$$

$$p_\nu = (1-\nu/n)N \qquad \nu=1,\ldots,n-1. \tag{22}$$

The $t_1(z)$ is given by solving the recurrence relations (21). All z_s^i satisfy the equations

$$t_\mu(z_u^{\mu+1}) = 0 \qquad u=1,\ldots,p_{\mu+1}; \quad \mu=1,\ldots,n-2. \tag{23}$$

Equations (20) and (23) are the Bethe ansatz equations of the $\mathbb{Z}_n \times \mathbb{Z}_n$ symmetric model of Belavin. The roots of the Bethe ansatz equations determine the eigenvalues $t_0(z)$ of the transfer matrix $t(z)$.

For a special case with $n=2$ our results coincide with those in the remarkable work by Takhtadzhan and Faddeev (1979). Further discussions and details of this letter will be published elsewhere.

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